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A UNILATERAL REPRESENTATION FOR AUTOREGRESSIVE RANDOM FIELD MOD--ETC(U)

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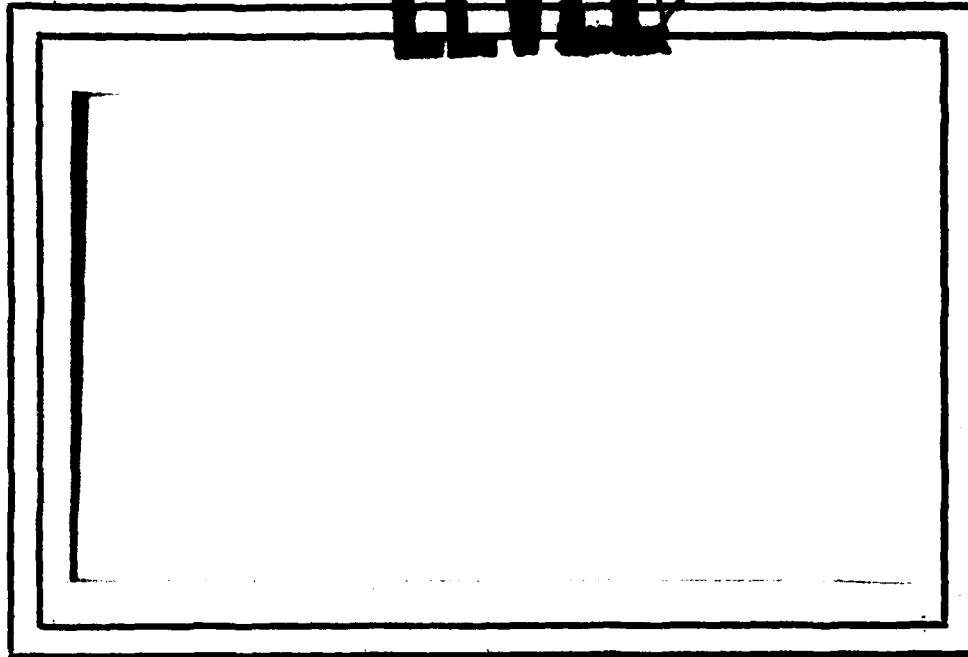
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6 A UNILATERAL REPRESENTATION FOR
AUTOREGRESSIVE RANDOM FIELD MODELS.

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ABSTRACT

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1. Introduction

In this paper we shall deal with a subset of stationary (wide sense) processes with absolutely continuous spectral distributions which are rational functions of the two quantities $e^{i\theta_1}, e^{i\theta_2}$. More precisely we shall study the process $X_{[m,n]} \in \mathbb{R}^d, [m,n] \in \mathbb{Z} \times \mathbb{Z}$ on an infinite lattice, with covariance structure

$$E(X_{[m+s,n+t]} X_{[m,n]}^*) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-is\theta_1 - it\theta_2} f(\theta_1, \theta_2) d\theta_1 d\theta_2, \quad (1.1)$$

and zero mean.

We assume $f(\theta_1, \theta_2)^{-1}$ exists and is finite at every (θ_1, θ_2) , and

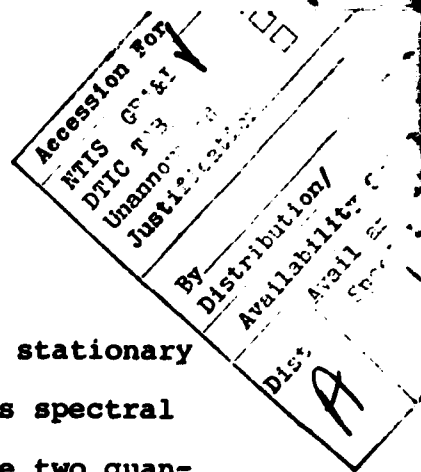
$$f(\theta_1, \theta_2)^{-1} = (a_{[0,0]} + \sum_{[m_1, m_2] \in N^p} a_{[m_1, m_2]} \cos(m_1\theta_1 + m_2\theta_2)) \quad (1.2)$$

$a_{[0,0]}, a_{[m_1, m_2]}$ are $p \times p$ matrices satisfying $a_{[s,t]}^* = a_{[-s, -t]}$. V^* is the complex conjugate transpose of the vector V . N^p

denotes the deleted $p \times p$ neighborhood of $[0,0]$, that is,

$$\{[m_1, m_2] : |m_1| \leq p, |m_2| \leq p, [m_1, m_2] \neq [0,0]\}.$$

Models of this type have been used as models of texture images [1,2]. In the case where $X_{[...]}$ is a Gaussian process, it can be shown [3] that $X_{[...]}$ is a Gauss-Markov process with respect to N^p ; that is,



$$\begin{aligned} \text{Prob}(X_{[m,n]} | X_{[s,t]}, [s,t] \neq [m,n]) = \\ \text{Prob}(X_{[m,n]} | X_{[s,t]}, [s,t] \in [m,n] + N^P) \end{aligned} \quad (1.3)$$

In fact, the process with spectral density $f(\theta_1, \theta_2)$ satisfies the conditional model

$$\begin{aligned} E(X_{[m,n]} | X_{[s,t]}, [s,t] \neq [m,n]) = \\ -a_{[0,0]}^{-1} \left(\sum_{[m_1, m_2] \in N^P} a_{[m_1, m_2]} X_{[m-m_1, n-m_2]} \right) \end{aligned} \quad (1.4)$$

Conditional models of this type have been found useful in the modeling of spatial patterns [7]. It is also known (see, for example, page 26 of [7]) that no finite one-sided representation for this model exists of the type (with S finite subset of $\mathbb{Z} \times \mathbb{Z}$)

$$b_{[0,0]} X_{[m,n]} + \sum_{[m_1, m_2] \in S} b_{[m_1, m_2]} X_{[m-m_1, n-m_2]} = Z_{[m,n]}$$

where $Z_{[m,n]}$ is a process of uncorrelated noise.

The purpose of this paper is to show that the collection of spectral representations of the process $X_{[...]}$ along one of the coordinates is representable as a one-sided finite order "time series" model along the other coordinate. Thus, in this sense it is seen that all ARF's have a "causal" representation.

This method of producing a one-sided representation can be contrasted with the so-called NSHP (non-symmetric half plane) representation of [3] and [6].

2. A Unilateral Representation

We consider the process $X_{[...]}$ with spectral density

$$f(\theta_1, \theta_2) = [a_{[0,0]} + \sum_{[m_1, m_2] \in \mathbb{N}^p} a_{[m_1, m_2]} \cos(m_1 \theta_1 + m_2 \theta_2)]^{-1}, \quad (2.1)$$

which is a p -th order autoregressive process.

Let $z = e^{-i\theta_1}$, $w = e^{-i\theta_2}$, and rewrite the above equality as

$$f(\theta_1, \theta_2)^{-1} = a_0(w) + a_1(w)z + \dots + a_p(w)z^p + a_1^*(w)z^{-1} + \dots + a_p^*(w)z^{-p}.$$

For each fixed w we can consider $f(\theta_1, \theta_2)$ as a spectral density in θ_1 . We next produce a causal factorization of $f(\theta_1, \theta_2)$ in the form

$$f(\theta_1, \theta_2)^{-1} = [c_0^*(w) + c_1^*(w)z^{-1} + \dots + c_p^*(w)z^{-p}] [c_0(w) + c_1(w)z + \dots + c_p(w)z^p], \quad (2.2)$$

where, for each $w = e^{-i\theta_2}$, $c_0(w) + c_1(w)\xi + \dots + c_p(w)\xi^p$ has no roots inside and on the complex unit circle $|\xi|=1$. ([4], page 65).

We next consider the spectral representation of the process $X_{[...]}$ along the second coordinate:

$$X_{[n,m]} = \int_{-\pi}^{\pi} e^{im\theta} dY_n(\theta), \quad (2.3)$$

where $Y_n(\theta)$ is the spectral representation of the process $X_{[n,...]}$. ([5], page 481).

Next expand each of $c_0(w), \dots, c_p(w)$ in a Fourier expansion

$$c_j(w) = \sum_{k=-\infty}^{\infty} e^{ik\theta} \sum_{j=0}^p \hat{c}_{[j,k]}.$$

Then ([4], page 61) the process satisfies the autoregression

$$\sum_{j=0}^p \sum_{k=-\infty}^{\infty} \hat{c}_{[j,k]} X_{[n-j, m+k]} = Z_{[n, m]} \quad (2.4)$$

where $Z_{[.,.]}$ is an uncorrelated white noise process. Let $W_n(\theta)$ be the spectral representation of the process $Z_{[n,.]}$:

$$Z_{[n, m]} = \int_{-\pi}^{\pi} e^{im\theta} dW_n(\theta). \quad (2.5)$$

We conclude with the following

Theorem: Let $\{Y_n(\theta)\}$, $\{W_n(\theta)\}$ be the spectral representations of the processes defined above. They satisfy the stochastic differential equation

$$\sum_{k=0}^p c_k(e^{-i\theta}) dY_{n-k}(\theta) = dW_n(\theta) \quad (2.6)$$

Proof: In the above autoregressive representation we substitute the spectral integrals and get (after combining terms)

$$v_m: \int_{-\pi}^{\pi} \left\{ \sum_{j=0}^p \sum_{k=-\infty}^{\infty} \hat{c}_{[j,k]} e^{i(m+k)\theta} dY_{n-j}(\theta) - e^{im\theta} dW_n(\theta) \right\} = 0$$

Factoring out $e^{im\theta}$ we have

$$v_m: \int_{-\pi}^{\pi} e^{im\theta} \left\{ \sum_{j=0}^p \sum_{k=-\infty}^{\infty} \hat{c}_{[j,k]} e^{-ik\theta} dY_{n-j}(\theta) - dW_n(\theta) \right\} = 0$$

As any continuous function $f(\theta)$, $\theta \in [-\pi, \pi]$ can be approximated in mean square by linear combinations of $e^{im\theta}$, the result follows.

3. The finite version.

The above calculation can be carried out in the case where we have a finite number of values

$$X_{[n,0]}, \dots, X_{[n,M-1]}$$

in the vertical direction.

Let $\psi_M = e^{2\pi i/M}$. The finite versions of the above spectral representations are as follows:

$$X_{[n,m]} = \sum_{k=0}^{M-1} \psi_M^{km} Y(n,k) \quad (3.1)$$

or

$$\Delta Y(n,k) = \frac{1}{M} \sum_{j=0}^{M-1} \psi_M^{-km} X_{[n,j]}. \quad (3.2)$$

That is, $\Delta Y(n, \cdot)$ is the finite Fourier transform of the data $X_{[n, \cdot]}$. Similarly

$$\Delta W(n,k) = \frac{1}{M} \sum_{j=0}^{M-1} \psi_M^{-km} Z_{[n,j]}. \quad (3.3)$$

The finite analogue of the above Theorem is

$$\sum_{k=0}^{M-1} \psi_M^{mk} \left\{ \sum_{j=0}^p b_j (\psi_M^{-k}) \Delta Y(n-j,k) - \Delta W(n,k) \right\} = 0$$

concluding with

$$\sum_{j=0}^p b_j (\psi_M^{-k}) \Delta Y(n-j,k) = \Delta W(n,k). \quad (3.4)$$

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